Special Topics in Functional Analysis

The aim of this course is to present some basic results in Linear Functional Analysis and to illustrate their usefulness in connection with the elliptic problems.
Prerequisites: Linear Algebra. Advanced Calculus (including elements of Topology and Lebesgue Integral).

Syllabus

1. **Banach spaces and linear operators**. Basic results. The principle of uniform boundedness. The inversion of bounded linear operators. The spectrum and the resolvent. Compact operators. Integral operators.
2. **Hahn-Banach theorem. Consequences**. Convex sets. The Hahn-Banach extension theorem. Locally convex spaces. Weak topologies. The theorem of Alaoglu-Bourbaki. Separation theorems. Reflexive spaces. The Krein-Milman theorem.
3. **Fixed point theorems**. The theorems of Banach, Schauder and Kakutani. Applications.
4. **Hilbert spaces and orthogonal expansions**. Hilbert basis. The orthogonal projection. The adjoint of an operator. Diagonalization of compact self-adjoint operators. Applications to Sturm-Liouville problems.
5. **Fourier transform**.
6. **Sobolev spaces and applications to PDE.**
7. **Rearrangements.** Schwarz symmetrization. The isoperimetric inequality. Applications.
8. **Unbounded operators**. Symmetric and self-adjoint operators. Semigroups of operators. The Hille-Yosida theorem.

Bibliography

1. H. W. Alt, *Lineare Funktionalanalysis*, Springer-Lehrbuch, Berlin, 1992.
2. J. B. Conway, *A Course in Functional Analysis*, 2nd ed., Springer-Verlag, Berlin, 1997.
3. C. P. Niculescu, *Lecture Notes in Functional Analysis*, Lahore, 2006.
4. W. Rudin, *Analyse fonctionelle*, Ed. Ediscience International, 1995.
5. K. Yosida, *Functional Analysis*, 5th ed., Springer-Verlag, Berlin, 1995.
6. M. Willem, *Analyse fonctionnelle élémentaire*, Ed. Cassini, Paris, 2003.

Instructor: Constantin P. Niculescu